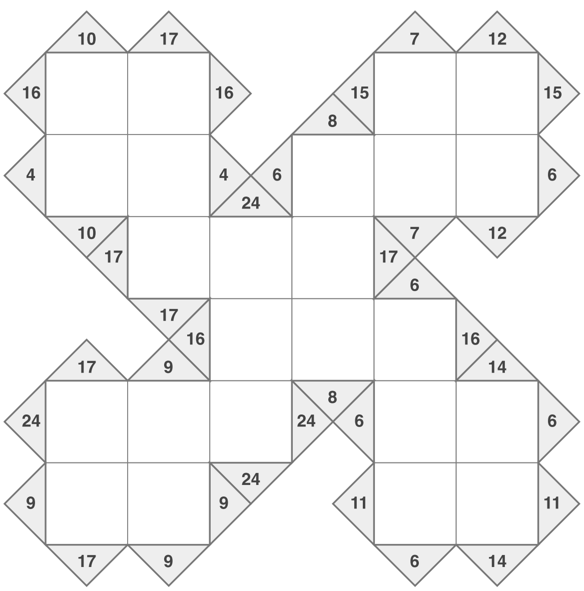
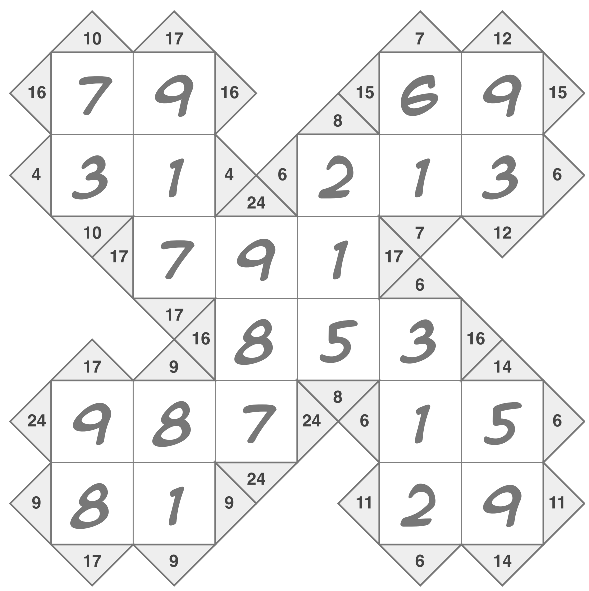
ABOUT THESE PUZZLES

Kakuro (カックロ literally “addition cross”), are a kind of numeric crossword puzzle, originally published as “Cross Sums” in the United States by Dell Magazines. They are now quite popular in Japan, and like Sudoku before it, that popularity has swept back to Europe and the United States. On my Krazydad website, Kakuro are the second most popular puzzle variety after Sudoku and its variants.



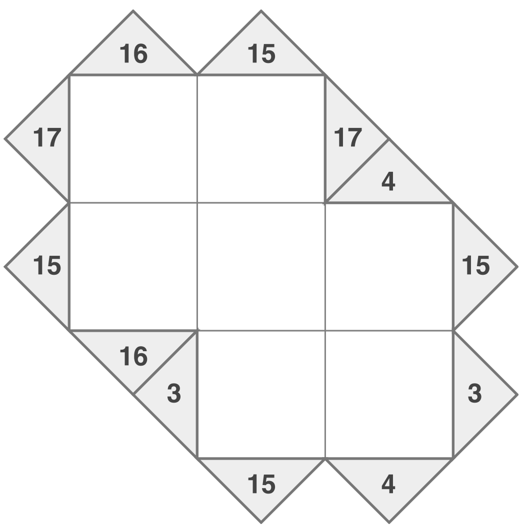


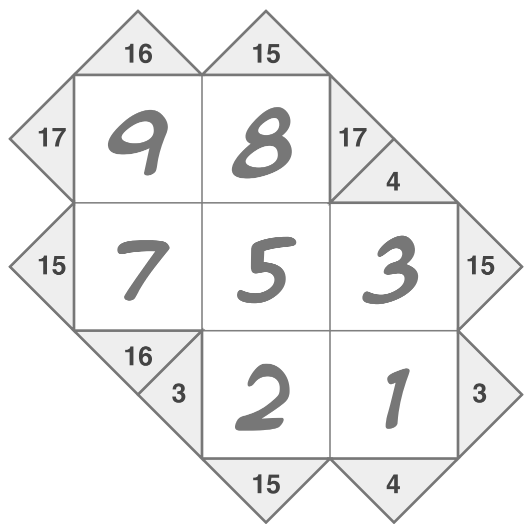
Since 2006, I have been publishing Kakuro in a unique format that is a little different from the norm - instead of using a rectangular grid with an extra row and column for clues, I like to preserve the visual symmetry of the puzzles, by employing dual triangular clues. I hope you'll agree that this makes the puzzles more attractive to look at, and the clues easier to spot.

THE RULES OF KAKURO

A Kakuro is made up of *sets** which are analogous to words in a normal Crossword. Each set is from 2 to 9 cells in length, and each cell (or square) in the set contains a digit, from 1 to 9. The triangular clues on both sides of the set indicate the sum you get when you add the digits together.

Digits may not repeat within a set. For example, if the clue on a set says 4, then it may contain 1 and 3 (in either order), but it may not contain 2 and 2. Likewise, if a set is nine cells in length, it must contain all the available digits 1 though 9 (and the clue will always be 45).

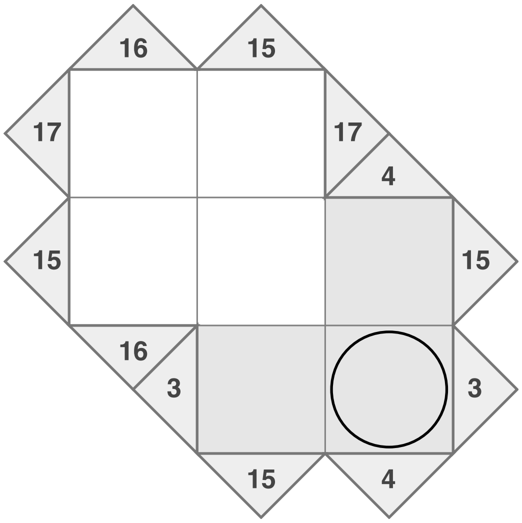




HOW TO SOLVE KAKURO

Let's start with the simple puzzle shown above. On the following pages, we'll solve it together.

* Puzzle nomenclature is not standardized. Other authors may use any of the terms *entry, block, container* or *cage* to refer to what I'm choosing to call a *set*.

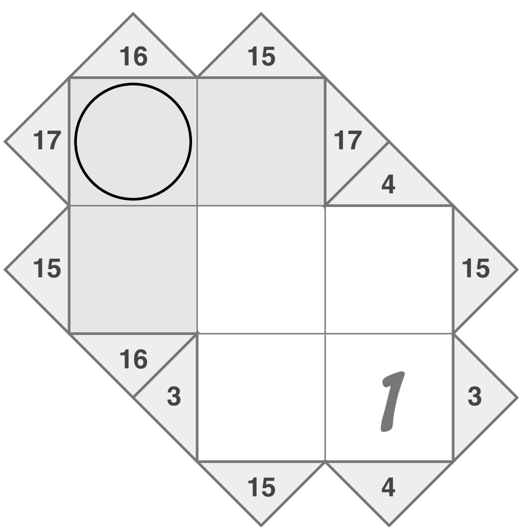


Restricted Sets

First look at the bottom two cells, which are clued 3, meaning the two digits must add up to 3. There are only two possibilities for each cell – 1 and 2. We don't know what order they appear, but we do know that only those two digits can be used. We call this a restricted set.

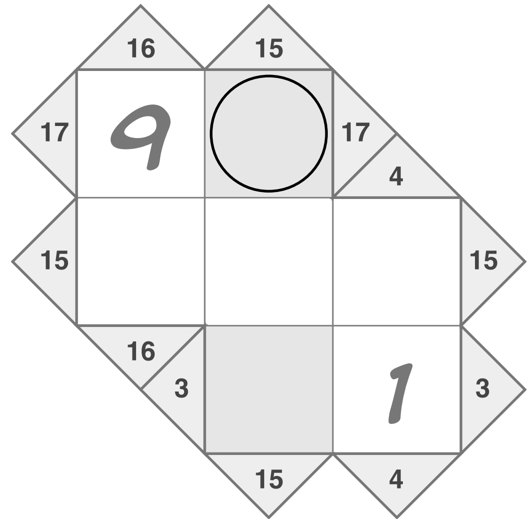
There's another restricted set nearby. On the right side of the puzzle, the set that sums to 4 can only contain 1 and 3 (remember that we can't use 2, since (2,2) would cause a repeating digit in the set, which is against the rules.

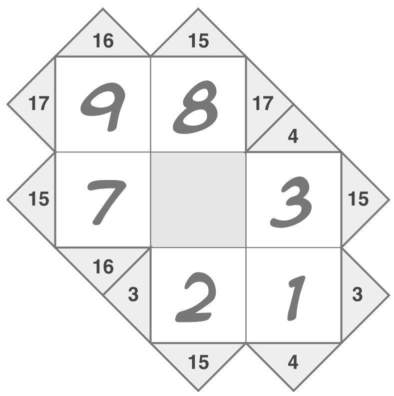
Because the two restricted sets cross each other, the digit in the bottom right corner has to be 1 – the only number common to both sets. When two restricted sets cross like this it often gives us a toehold to begin solving the puzzle.

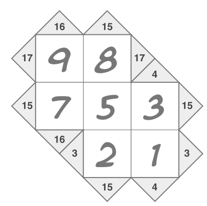


There are two more restricted sets on the upper left side of the puzzle. Again, we can solve the cell where they cross. The topmost set, clued 17, can only contain 9 and 8, while the leftmost set, clued 16, can only contain 9 and 7 (because 8+8 would be repeating digits). This means the digit on the top left-hand corner, where the two sets cross, must be the shared 9.

Now that we've solved the corners, we can fill in the rest of the two-cell sets using basic arithmetic. For example, the circled cell to the right of the 9 must be (17-9) or 8. Continuing in this fashion leaves only one unsolved cell in the middle, which you can easily figure out as 15-(7+3) or 5.







Length of Set	Clue	Possible Digits
2	3	1, 2
2	4	1, 3
2	16	9, 7
2	17	9, 8
3	6	1, 2, 3
3	7	1, 2, 4
3	23	9, 8, 6
3	24	9, 8, 7
4	10	1, 2, 3, 4
4	11	1, 2, 3, 5
4	29	9, 8, 7, 5
4	30	9, 8, 7, 6
5	15	1, 2, 3, 4, 5
5	16	1, 2, 3, 4, 6
5	34	9, 8, 7, 6, 4
5	35	9, 8, 7, 6, 5
6	21	1, 2, 3, 4, 5, 6
6	22	1, 2, 3, 4, 5, 7
6	38	9, 8, 7, 6, 5, 3
6	39	9, 8, 7, 6, 5, 4
7	28	1, 2, 3, 4, 5, 6, 7
7	29	1, 2, 3, 4, 5, 6, 8
7	41	9, 8, 7, 6, 5, 4, 2
7	42	9, 8, 7, 6, 5, 4, 3

As you can see, restricted sets are a very useful tool for solving Kakuro!

Here is a table, to use as a reference, which shows all the restricted sets with a length of 7 or less.

If you examine the table carefully, you'll notice a pattern to it:

There are four restricted sets for each set length – two for the smallest possible sums, and two for the highest.

This pattern changes when you get up to set lengths of 8 and 9.

We'll examine those on the next page.

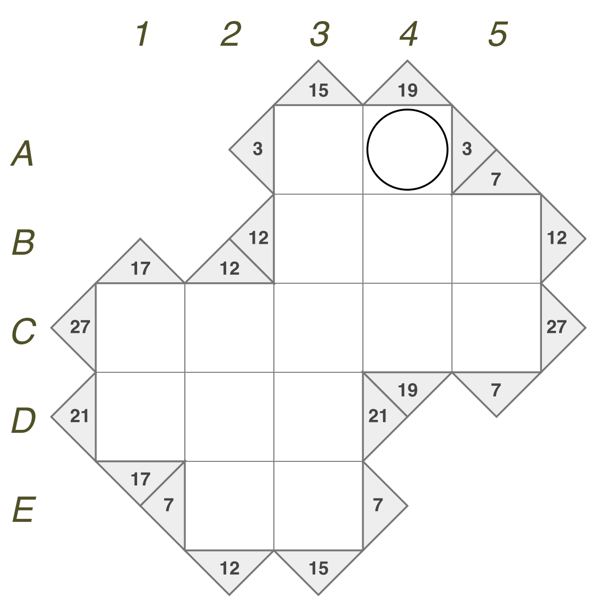
Length of Set	Clue	Possible Digits
8	36	1, 2, 3, 4, 5, 6, 7, 8
8	37	1, 2, 3, 4, 5, 6, 7, 9
8	38	1, 2, 3, 4, 5, 6, 8, 9
8	39	1, 2, 3, 4, 5, 7, 8, 9
8	40	1, 2, 3, 4, 6, 7, 8, 9
8	41	1, 2, 3, 5, 6, 7, 8, 9
8	42	1, 2, 4, 5, 6, 7, 8, 9
8	43	1, 3, 4, 5, 6, 7, 8, 9
8	44	2, 3, 4, 5, 6, 7, 8, 9
9	45	1, 2, 3, 4, 5, 6, 7, 8, 9

The sets that are 9 cells long are the simplest case. They must use all the digits, from 1 to 9, and they will always sum to 45.

The sets with a length of 8 are *all* restricted sets, and all are missing one digit. The missing digit can quickly be identifed as 45 minus the sum of the set. For example, for a set with a length of 8 and a clue of 42, the missing digit must be 45 - 42 which is 3.

You can use this same principle for other long sets as well. The missing digits will always add up to 45 minus the sum. So, for example, in a set with a length of 7 and a clue of 40, you'll know that the two missing digits must add up to 5, and that they must be either (2,3) or (1,4). Hence the remaining digits 5, 6, 7, 8 and 9 must appear somewhere in the set.

Now let's solve the larger puzzle, shown below, to see what other strategies come into play. This time I've added labels on the rows and columns to make it easier to identify individual cells.



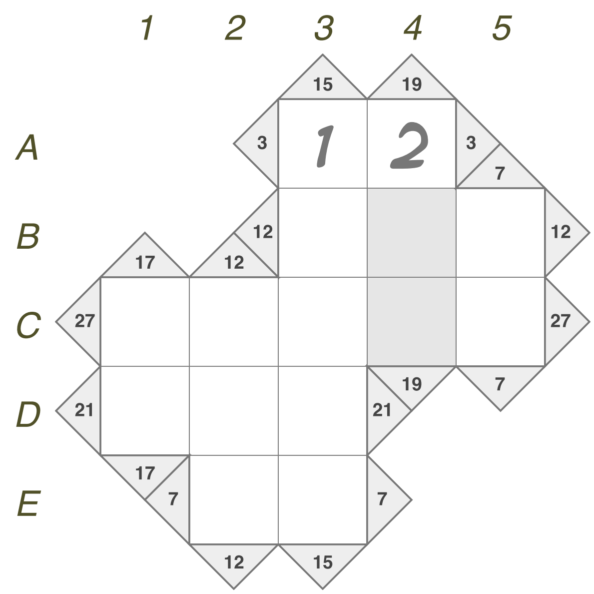
First let's look for restricted sets. You'll notice that there are a couple of them (clued 3 and 17), but that they don't cross each other. What to do?

Eliminating the Impossible

Let's start by examining cell *A4*. Notice that because it's part of a resticted set, clued 3, it can only have the value of 1 or 2. Consider what would happen if it had a value of 1. If so, the remaining two cells in the crossing set, clued 19, would sum to 18. The only two digits that can sum to 18 are 9 and 9, which would cause a repeating digit. 17 is the highest sum you can have for any two cells in a set. Therefore *A4* can't be 1. *A3* must be 1. A4 must be 2.

We know that the remaining two cells in the set clued 19, (*B4*, *C4*), must sum to 17, so these remaining cells form a restricted set of their own, and each one must be either 9 or 8.

We can write those in as possibilities. We'll also write them in as possibilities for the set clued 17 (*C1*, *D1*).

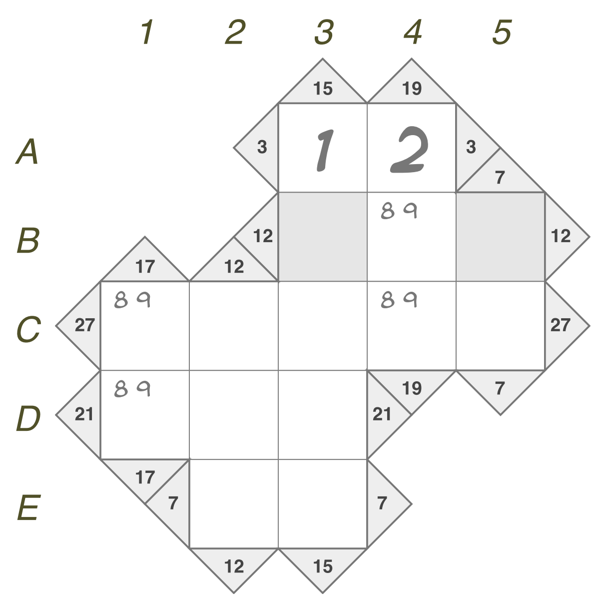


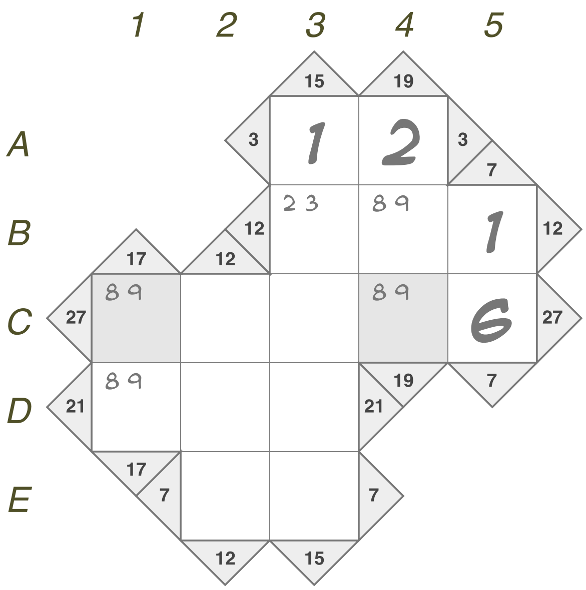
At this point it looks like we've run out of restricted sets, or have we?

Take a look at the set clued 12 that runs from *B3* to *B5*. We know that the middle cell, *B4*, must be 8 or 9. This means the remaining two cells must sum to either 12-8 or 12-9, in other words, 4 or 3, both of which make restricted pairs! This means that one of those cells is going to be a 1, and the other cell will be either a 2 or a 3. So which cell is going to be the 1?

It can't be *B3*, because it is part of a crossing set that already contains a 1. So the 1 must go into *B5*, and we can put 2 and 3 as possibilities for *B3*.

We can also finish the 7-down by putting a 6 in *C5*.



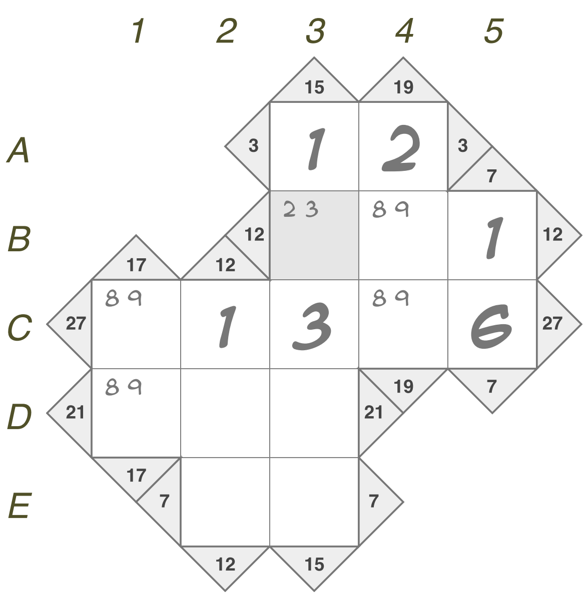


Naked Subsets

Now take a look at cells ***C1*** and *C4*, which are part of the longer set clued 27 which runs from *C1* to *C5*. Notice that both cells have the same two possible numbers: 8 and 9. When this happens it is called a naked subset (it can also happen with 3 digits in 3 cells, and so on). We know that one of these cells must be 8, and the other one must be 9. We just don't know which is which.

We also know that the two cells add up to 17. Since the 6 in that set is also filled in, we know that the three cells (*C1, C4, C5*) add up to 23, and we know that the remaining two cells must add up to 27 - 23 or 4. This means that (*C2, C3*) form a restricted set with the only possible values 1 and 3.

Once again, the 1 in *A3* prevents us from putting a 1 in *C3* (since they are part of the same set), so the 1 must go in *C2*, and the 3 must go in *C3*.

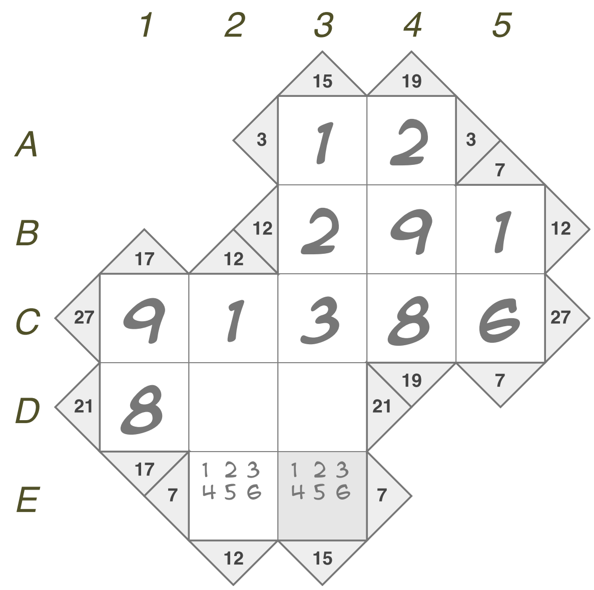


Now that we've solved *C3*, we've eliminated 3 from *B3*, which sets off a whole chain of easy solutions using simple arithmetic:

B3 = 2
B4 = 9
C4 = 8
C1 = 9
D1 = 8

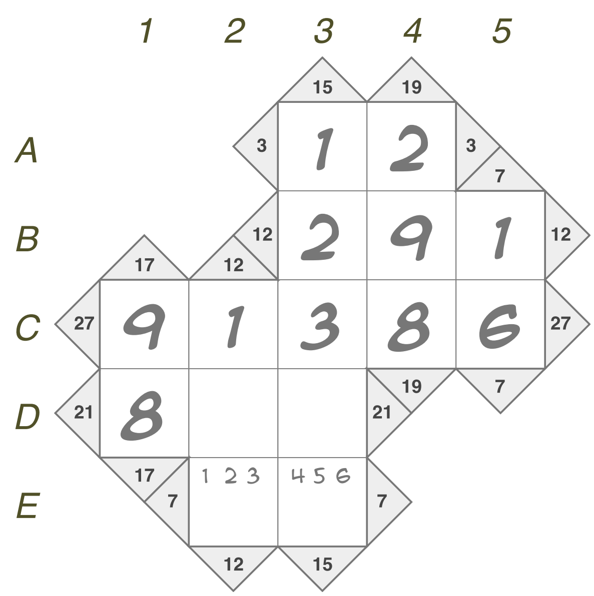
Matching Pairs

Now there are only four cells left, but we seem to be out of easy solutions. Let's write in the possible digits that remain. Notice that in the set clued 7 on the bottom, no digit can be larger than 6, so let's write those in.

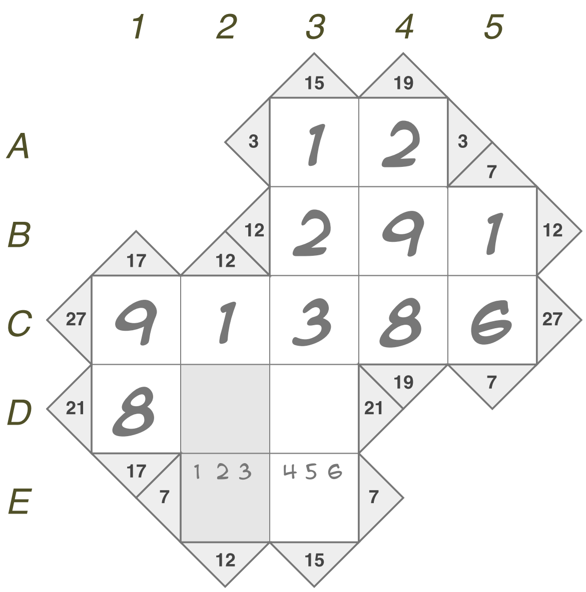


Also notice that we can’t use the 1,2 or 3 in cell *E3* (because they are already in use in the crossing set). This means that we also can’t use the corresponding 6,5 or 4 in cell *E2* (the complimentary numbers needed to add up to 7).

This principle of eliminating complimentary numbers is called matching pairs.



In this remaining part of the puzzle, we have 4 interlocking pairs. The matching pair technique will let us complete the puzzle.



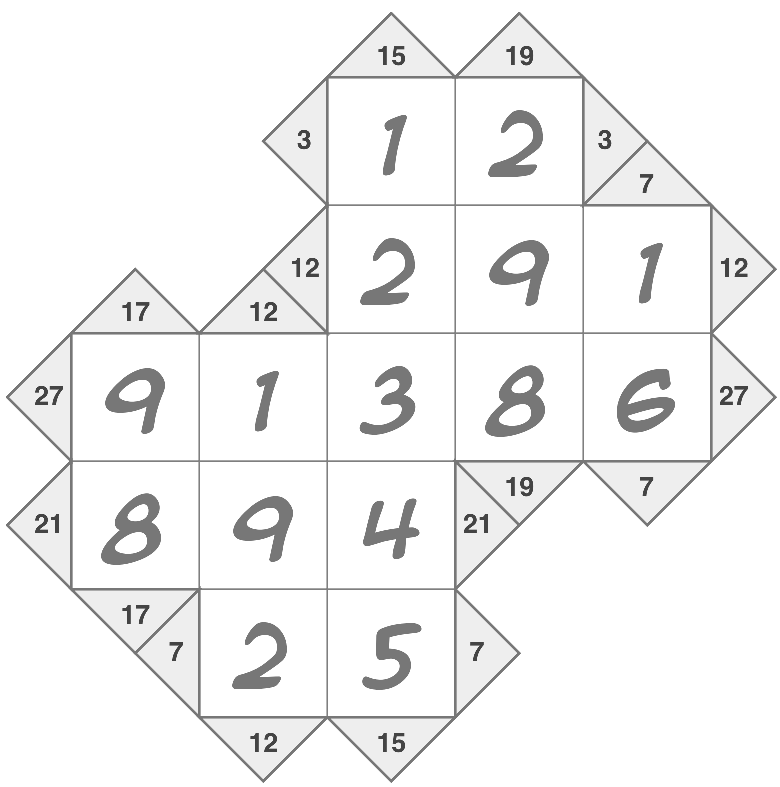
Take a look at the *D2*/*E2* pair. We know that these two cells must sum to 12 - 1 or 11. This means the possible values for *D2* are 10 (an illegal value), 9, or 8 (a duplicate with *D1*). The only legal value for *D2* is 9.

The remaining cells are easily solved:

E2 = 2
E3 = 5
D3 = 4

Solved it!

In the next section, we'll go over some techniques that are useful for more intractable puzzles.

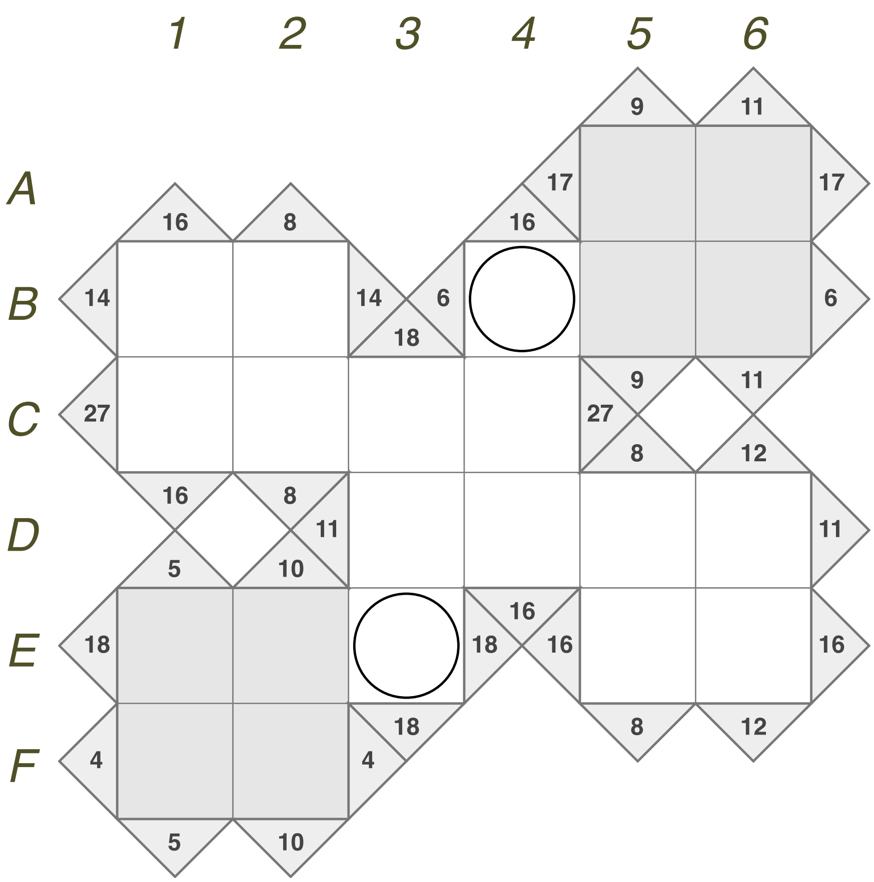


ADVANCED STRATEGIES

Criss-cross Arithmetic

You will occasionally encounter a puzzle in which a few isolated cells can be determined right away by adding or subtracting clues at the corners.

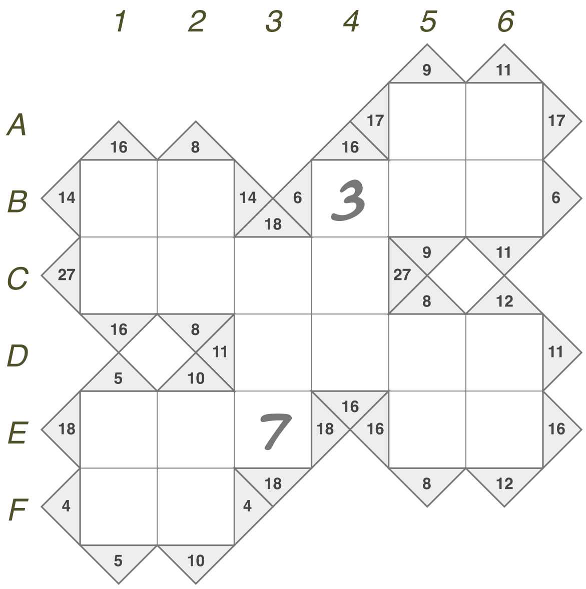
Consider the following puzzle:



You can figure out the value of *B4* by adding two horizontal sets (17+6) and subtracting the two crossing vertical sets (9+11) which share all but one cell, *B4*. (17+6)-(9+11) = 23-20 = 3

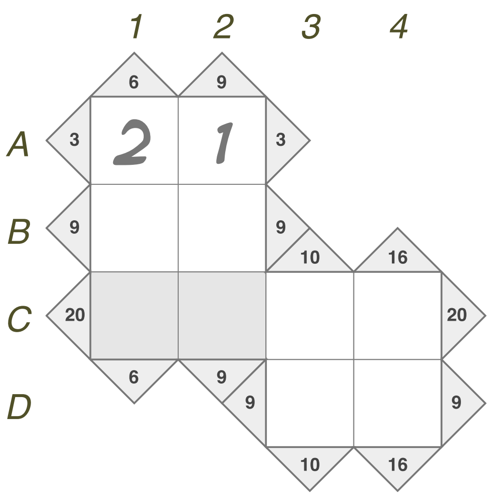
So *B4*=3.

I call this technique criss-cross arithmetic. Try using it to determine the value of *E3*.



$E3 = (18 + 4) - (5 + 10) = 7$

Even when you can't use criss-cross arithmetic to determine the value of an individual cell, you can often use it to determine the sum of pairs or triplets, which are part of larger sets. If those subsets have restricted sums, you can exploit them.



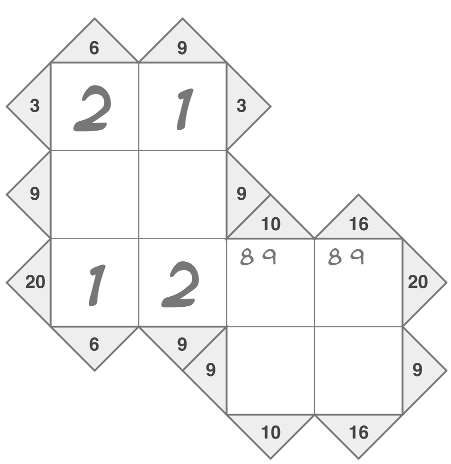
Consider the partially solved puzzle on the left.

Criss-cross arithmetic tells us that cells *C1* + *C2* must sum to a value of

$$(6 + 9) - (3 + 9) = 3$$

This means those two cells must contain a 1 and 2.

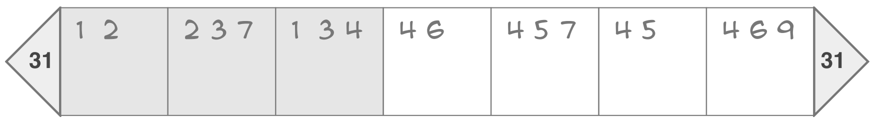
Since a 1 and 2 are already filled in in the crossing sets, we know where the new 1 and 2 must go.



Also, since *C1* + *C2* = 3, we know the two remaining cells in the set, *C3* + *C4* must contain 8 and 9 (because 20 – 3 = 17).

Hidden Subsets

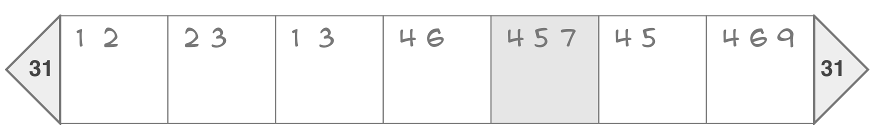
Sometimes you will encounter a longer set in which there is a pair or triplet which contains a specific number of possibilities that aren't found in the other cells. Consider the following set, which we've already narrowed down to the following possibilities:



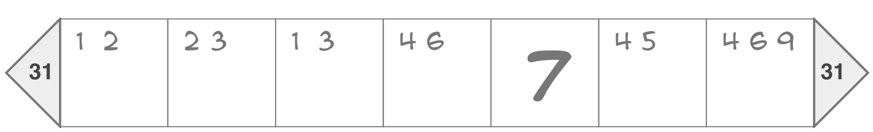
In this set the first three cells contain 1, 2 or 3 as possibilities, but those digits aren't found in any of the other cells. If we knew for a certainty that 1, 2 and 3 were required for this set, then we would also know that those 3 cells must contain the 1, 2 and 3, and we could eliminate other numbers as possibilities for those first three cells, converting them into a naked triplet.

It turns out that 1, 2 and 3 are indeed required! We know this because the sum of the set is 31. The missing two digits must sum to 45 - 31 = 14, and therefore neither of the missing digits can be smaller than 5. Since 1, 2 and 3 are required in the set, the first three cells form a hidden subset. We can eliminate all the other numbers except 1, 2 and 3 from the first three cells.

After eliminating these, you'll notice there is only one remaining cell with 7 as a possibility:



Again, we know that 7 is *required* in this set, because the missing two digits sum to 14. Since 7+7=14 would cause repeating digits, 7 can't be one of the missing digits. 7 is required, and this must be the cell that holds it!



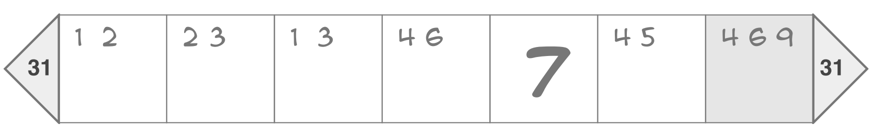
Binding Cells

You can constrain any cell to a minimum and maximum value. This is called establishing the bounds, or binding the cell, and it can be very useful on otherwise unwieldy sets.

To determine the maximum value of any cell, figure out the smallest possible sum you can form from the remaining cells and subtract this value from the set's clue. We know the cell can't get any larger than this value, and any digits that are larger than this number can be eliminated.

Similarly, to figure out the minimum value of any cell, figure out the largest possible sum you can form from the remaining cells and subtract this value from the set's clue. We know the cell can't get any smaller than this, and any digits that are smaller than this number can also be eliminated.

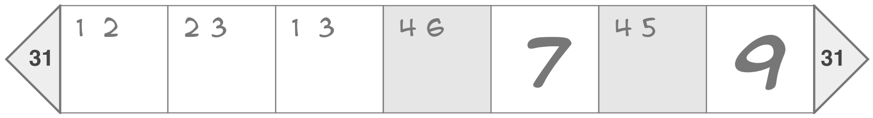
The best candidates to try the binding technique on first are the ones with the widest range of possible values. This is because the remaining cells will tend to have the narrowest range of possible sums. Consider the set that we have been working on:



The cell with the widest range of possible values is the rightmost one.

The smallest sum we can form from the remaining cells is 1+2+3+4+7+5 = 22. The largest sum we can form from the remaining cells is 1+2+3+6+7+5 = 24. So the maximum possible value for the rightmost cell is 31 - 22, or 9. The smallest possible value is 31 - 24, or 7.

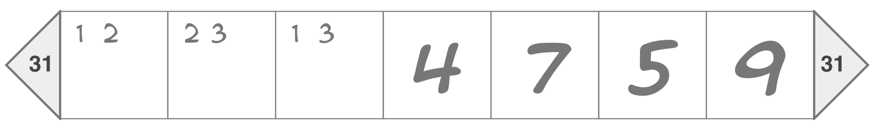
Since 7 is the smallest possible value, we can eliminate 4 and 6 as possibilities, leaving only 9.



Now that we've found the 9, we can also determine the values of two more cells.

We already know that the set contains 1, 2, 3, 7 and 9. Adding these numbers up gives us 22.

This means that the two shaded cells above must contain a number which sums to 31 - 22, or 9. The only available values that work are 4 in the first cell and 5 in the second cell.



Since this is just a fragment, we can't completely solve the first 3 cells, but as you can see, we've accomplished quite a lot without a whole lot to go on.

So ends our discussion of advanced techniques. As you solve more and more of these puzzles, you will discover a few more. Be sure to check out my website and blog at krazydad.com for more tips and advice.

You can always email me at dad@krazydad.com if you have any questions about puzzle solving or if you get stuck.

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