ADVANCED SUDOKU

The Golden Chain Technique

The Golden Chain technique is an enhancement of the XY-Wing as described by Robert Woodhead in the document included in his Sudoku Susser program (http://www.madoverlord.com/projects/sudoku.t) Throughout this document, I use similar definitions as in Robert's document. So, please, make sure you have a copy of such as reference.

The Golden Chain technique is extremely powerful and very easy to use as you will see, and it will help you solve the most difficult puzzles without the need of guessing or complicated tabling techniques.

First of all, we need some definitions:

__**Link:**__ Two unresolved cells are a "link" if:

1. they both are in the same block, row or column,
2. each cell has only two possible answers, and
3. they share at least one possible answer.

Notation: if c1 has possibilities (XY) and c2 has possibilities (YZ), we will refer to the link as **X (c1) Y (c2) Z**. In this case, we call X and Z the "ends" and we call Y the "bridge".

Figure 1. Examples of links:

5	1	3	279	479	24	2679	8	467
7	2	6	5	1349	8	139	14	134
48	49	489	279	13479	6	12379	5	1347
68	67	78	1	2	5	4	3	9
9	3	12	4	6	7	8	12	5
124	47	5	3	8	9	127	6	17
1234	4579	12479	6	34579	234	13	14	8
346	4569	49	8	3459	1	36	7	2
12346	8	1247	27	347	234	5	9	1346

The blue lines in the puzzle give example of links. They can be written like this:

1. **8 (r3c1) 4 (r3c2) 9**
   This means that r3c1 and r3c2 form a link where 8 and 9 are the end values and 4 is the bridge.
2. **6 (r4c2) 7 (r6c2) 4**
   Again, r4c2 and r6c2 form a link with ends 6 and 4, and bridge 7.
3. A “naked pair is a very special kind of link that can generate two different links like this: **1 (r2c8) 4 (r7c8) 1**, or **4 (r2c8) 1 (r7c8) 4**.
4. A bridge can be diagonal with a block like this: **7 (r6c9) 1 (r5c8) 2**.

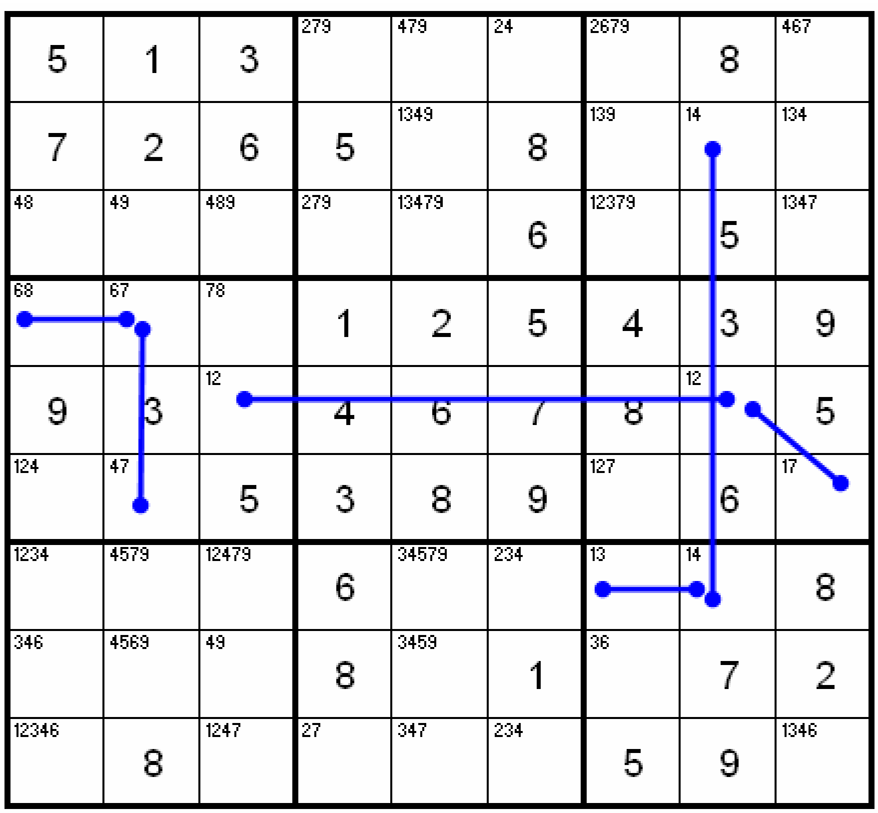
<u>**Consecutive Links:**</u> Three cells (c1, c2, c3) form “consecutive links” if:

1. c1 and c2 form a link,
2. c2 and c3 form a link, and
3. both links have different “bridges”

Notation: We can refer to a consecutive link by using the notation **X (c1) Y (c2) Z (c3) W**, where c1 has possibilities XY, c2 has possibilities YZ, c3 has possibilities ZW and Y is different from Z. Again, we will can X and W the “ends” and Y and Z the “bridges.”

Figure 2. Examples of Consecutive Links:

In the same puzzle above, we can construct a few consecutive links like this:



The consecutive links identified are:

1. **8 (r4c1) 6 (r4c2) 7 (r6c2) 4**
   whre 8 and 4 are the “ends”, and 6 and 7 are the “bridges”
2. **1 (r5c3) 2 (r5c8) 1 (r6c9) 7**
   Note that the naked pair form by r5c3 and r5c8 can generate two different link (with bridges 1 or 2), but the requirement that consecutive links cannot have the same bridge implies that we can only use one of them in this example.
3. **1 (r2c8) 4 (r7c8) 1 (r7c7) 3**.

We can now introduce the concept of a chain:

Chain: N+1 cells form a “N-link Chain” if they form consecutive links and each cell is at most part of two links.

Notation: We can refer to a chain by using the notation

A1 (c1) B1 (c2) B2 (c3) B3 **B_N (c_{N+1}) A2**,

where c1, c2, . . . , c_{N+1} are the cells, B1, B2, . . , B_N are the bridges, and A1 and A2 are the ends of the chain.

Comment: the requirement that “each cell is at most part of two links” is needed to avoid “loops” within the chain. You can refer to Robert’s document on how to make some good use of loops under other techniques)

Figure 3. Examples of Chains

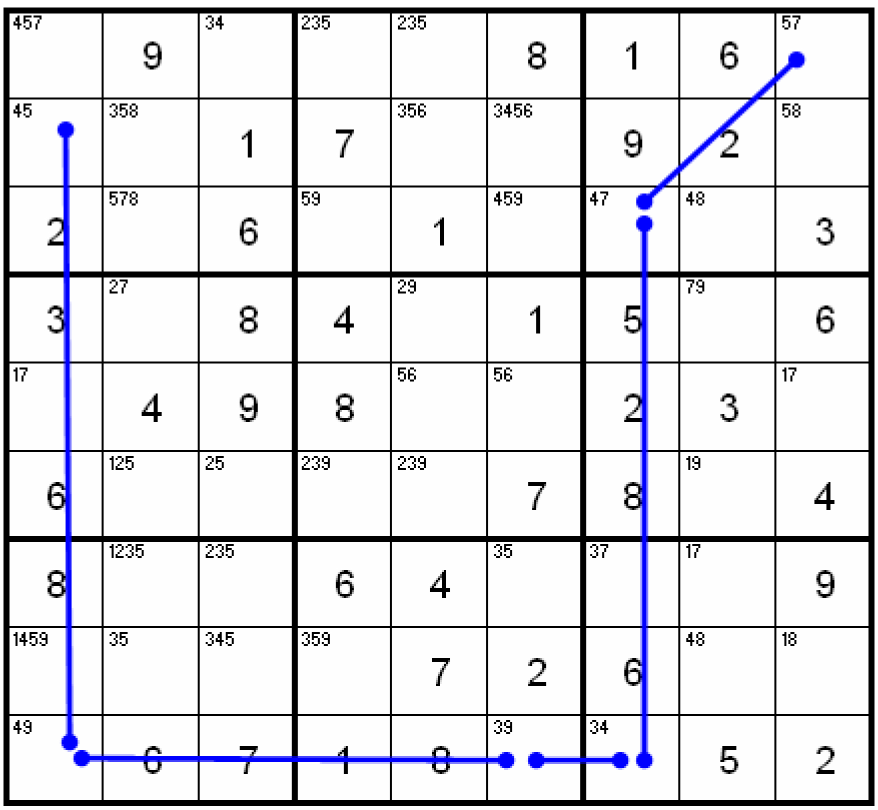
457	9	34	235	235	8	1	6	57
45	358	1	7	356	3456	9	2	58
2	578	6	59	1	459	47	48	3
3	27	8	4	29	1	5	79	6
17	4	9	8	56	56	2	3	17
6	125	25	239	239	7	8	19	4
8	1235	235	6	4	35	37	17	9
1459	35	345	359	7	2	6	48	18
49	6	7	1	8	39	34	5	2

The two chains identified can be written as:

1. **5 (r2c1) 4 (r9c1) 9 (r9c6) 3 (r9c7) 4 (r8c8)** 8
   with ends 5 and 8, and bridges 4, 9, 3, and 4. Note that “4” appears twice as a bridge, but not consecutively (which is the requirement).
2. **2 (r4c5) 9 (r4c8) 7 (r5c9) 1 (r6c8) 9**
   with ends 2 and 9 and bridges 9, 7 and 1.

Golden Chain: A “Golden Chain” is a chain where the two “ends” have the same value.

Figure 4. Examples of a Golden Chain



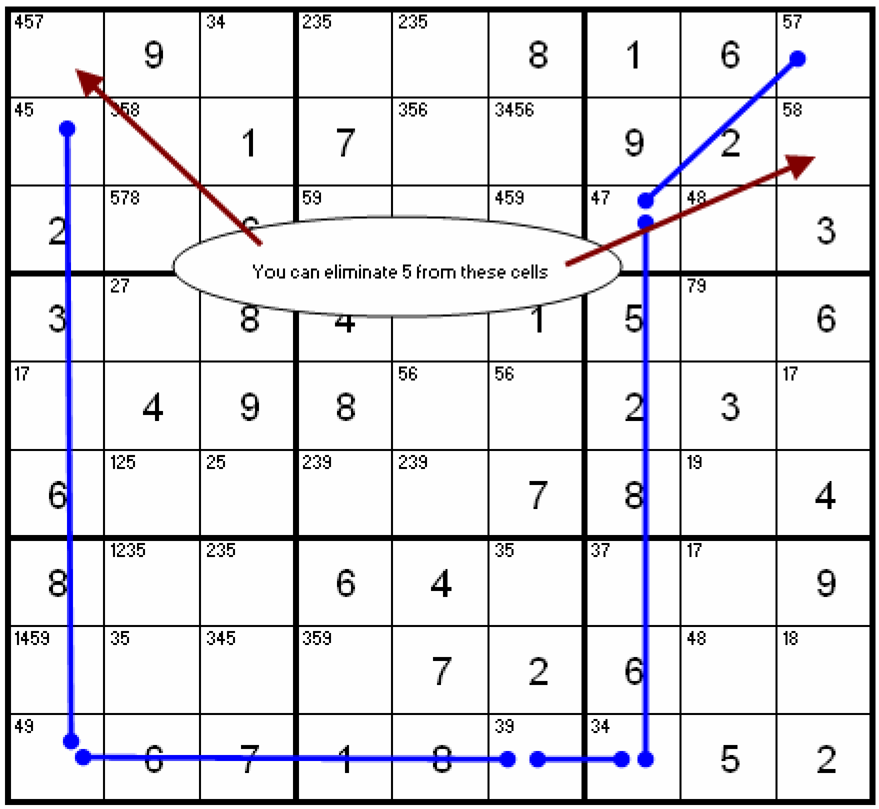
The Golden chain identified is: **5 (r2c1) 4 (r9c1) 9 (r9c6) 3 (r9c7) 4 (r3c7) 7 (r1c9) 5**. Note that both ends are equal to 5.

Here is the main THEOREM:

THEOREM *(“The Golden Chain Technique”):*If you have an N-link Golden Chain with end values equal to A, , then you can remove A from any cell that is a buddy of both c1 and c_{N+1} , where c1 and c_{N+1} are the first and last cells of the chain.

Example of the Golden Chain Technique:

In the same puzzle of figure 4, r1c1, r1c2 and r2c9 are “buddies” of the first and last cell of the Golden Chain. Therefore you can eliminate the “end” values from all of them. In this case you can eliminate 5 from r1c1 and r2c9.



PROOF OF THE THEOREM: You can see that if c1 is not equal to A, then it must be B1, then c2 must be B2, then c3 must be B3, etc; and c_{N+1} must be A. Therefore either c1 is A or c_{N+1} is A. This means that a cell that is a “buddy” of both c1 and c_{N+1} cannot be A.

Note: You can see that if the Golden Chain has only two links, then it becomes the same as an XY-Wing.

How to use this technique:

I have used this technique with repeated success by following the following strategy:

1. Use other deduction techniques to eliminate as many possibilities as you can so that you have plenty of cells with only two possibilities. In particular, Naked Sets, Hidden Naked Sets, Comprehensive Naked Sets, Comprehensive Hidden Sets, and X-Wings (as explained by Robert Woodland) are very effective to help you achieve this.

2. Once you have a good number of cells with two possibilities, you can build Golden Chain around your puzzle. The more you use this technique, the easier it is to quickly identify the ends of the Golden Chain that are of your advantage.

Final thoughts: I hope this technique helps you get a little more sleep when you are able to solve your puzzles in a shorter amount of time. GOOD LUCK.

For comments and suggestions about this technique, please email tech@sdne.com.

emc

January 5, 2006.